MATH MAGIC BLOCKS

Instruction manual

PATENT No. 8591234

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F o r w a r d

A happy alternative to force-feeding formulas

MMB's are a powerful learning tool to help children lay a strong foundation in basic math skills. Just snap the pieces of the precision2-pan balance together, present the blocks, and stand back. Watch children start UNlearning what, according to a recent study most students believe about math:

> *77% of the students seemed to believe that math was not something that could be figured out, or that made sense. It was just a set of procedures and rules to be memorized.*[1]

One student in the study spoke for many when he said, ""I don't think [being good at math] has anything to do with reasoning. It's all memorization."

OW! That hurts. No wonder a majority of the 120 students I polled said math was their most hated class. But math is beautiful! Math is fun. Math Magic Blocks helps kids realize this. Once, driven by curiosity instead of anxiety, they develop the confidence that they can do math, competence follows, laying a strong foundation for success in higher grades. Replace memorizing steps with understanding. Kids don't have to be forced to learn math. With MMB's they want to.

The Math Magic Blocks philosophy fits hand-in-glove with a program at Stanford University, developed by Jo Boaler, a professor of Math Education. Her results are incredible–and hopeful. To change *your* attitude about teaching math, go to youcubed.org and scroll down to the 3-minute video (which shows Professor Boaler in pink shirt). To quote a student who took her summer math workshop:

> I thought that math was all about right answers and wrong. Now that I believe math is about ideas and creativity, it's a lot more inspiring.

Students who participated in the 18-lesson workshop improved their performance on standardized tests by an average of 50%, the equivalent of 2.4 years of school. If students like what they're learning, it lasts:

"Just as eating against one's will is injurious to health, so studying without a liking for it spoils the memory, and it retains nothing it takes in."

--L e o n a r d o d a V i n c i

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[1] (Richland, L. E., et. al. (2012). Teaching the conceptual structure of mathematics. Educational Psychologist, 47(3), 189–203)

PREFACE

WHY I MADE Math Magic Blocks

(Please be patient with my PATENT No. 8591234)

WHOA! Have you ever seen such a portentous patent number? Let me count the ways it proves 8591234 and MMBs were meant to be.

Kids first learn addition, so let's start there. The digits of my patent number add up to:

$$8 + 5 + 9 + 1 + 2 + 3 + 4 = 32$$

Note that the fifth digit is 2, the second digit is 5.
And what is 2 raised to the 5^{th} power (2^5)?

$$2\times2\times2\times2\times2=32$$

Of course the third digit is 9, which is equal to 3^2, which you'd also have to write as 32 if your superscript function on your computer wasn't working. But there's even more to this mystical 32.

After addition with MMB's comes multiplication. So lets do both:

$$8\times5\times9\times1\times2\times3\times4 = 8640$$

If we now add those digits: 8+6+4+0 = 18

A 2-pan balance, with a central pedestal, is part of MMB's. How interesting that the central digit in the patent is 1, which looks quite a bit like a central point pedestal! Moving out in both directions from this central point pedestal, we find 9 and 2. Note that:

$$9\times2 = 18$$

the very same number we got by multiplying and adding the patent digits.

MOST of the number between 1 and 60 are NOT included in the MMB box. Yet the two digits, moving out from the middle 1 are 5 and 3, whose product is 15, which is–you guessed it–is one of the blocks in the set. At the two ends of the patent number we find 8 and 4, whose product is:

$$8 \times 4 = 32 \text{ (once again)}$$

But that's just for starters! We continue to 859. The center point is 5, which equals 2 plus 3. But 2 *times* 3 equals 6.
Let us dwell (if the reader has a moment?), on the fact that the 5^{th} and 6^{th} numbers of the patent

are . . . 2 and 3. Thus:

$$2 + 3 = 5 \text{ (the black blocks add)}$$
$$2 \times 3 = 6 \text{ (the colored blocks multiply)}$$

Precious! But getting back to 859, we see 8 on the left of 5 and 9 on the right. Of course since we've just noted that 5 = 2 + 3, it is pretty hard to NOT notice that the 8, 9, 2, and 3 all tie together nicely this way:

$$8 = 2^3 \quad \text{and} \quad 9 = 3^2$$

But there is still more in the 859 to challenge those who would tell you that patent numbers are assigned in meaningless numerical order:
As we've tried addition, multiplication and exponentiation, let's see what subtraction turns up. The differences between the 5 center point and its flanking digits is thus:

$$8 - 5 = 3$$
$$9 - 5 = 4$$

You will note that aligning these two expressions:

$$\begin{array}{r} 8 - 5 = 3 \\ 9 - 5 = \underline{+\,4} \\ 7 \end{array}$$

yields both the highly mystical/Biblical 7 AND the total number of digits in the patent number. Note also that 4 is the ONLY number for which it can be stated that

$$4 = X + X = X \times X$$

This then reminds one of the beautiful connections already discovered for addition (5 = 2 + 3), and multiplication (6 = 2 × 3).

Returning to the mystical properties of 4, with a supporting role from 5 and 2, if we divide the first 5 digits by the last 2 (85912 ÷ 34) we get 2526.823529 as displayed on a student calculator. Without dallying to marvel how EVERY ONE of these digits has played a starring role in our analysis already, it will be noted that adding those digits together yields:

$$2+5+2+6+8+2+3+5+2+9 = 44,$$

a first two digits of the divine descant that harmonizes with this song:
If we take 859 and turn it into 85 ÷ 9, we get 9.4444444444. . .

Thus 9 pops up once again, and the 4? Its never ending echo reminds us of both the length of the linear numerical sequence in the patent number (1234) AND the final digit. Did we need further proof of the ineluctable connectedness of MMB's patent number?

Perhaps. Consider that moving the slant fraction line one place to the left (from 85912/34, to 8591/234), yields 36.71367. . . Here again, the "coincidences" strain belief:

1) Just as in the patent, the digit 1 occupies the center point.
2) Our mysterious friend, 7, makes double appearance.
3) Behold a double double– "36" also appears twice.
4) BTW, did you happen to notice exactly how many blocks are included in the Math Magic Blocks box. . .?

Note that, among those colored blocks, the smallest number is 2 and the largest number is 60. If we subtract 1 from each of these, we get 1 (the center point of the patent) and 59 –center point's left hand companions. This brings us back to center point's *right* hand companions, 2 and 3, which we've already discovered bear multiple interconnected mystical messages.

Though there are many other proofs that MMB's and patent 8591234 were destined to be conjoined, I will stop here, but not without first mentioning *why* the first few pages of this instruction manual have been wasted on such, er, pointless drivel.

I had fun driveling. Yes, ferreting out these seemingly improbable coincidences proved to be an entertaining mathematical recreation. It is certainly NOT a coincidence that "Mathematical Recreations" was the title of a column written by Ian Stewart for *Scientific American* for ten years, or that the marvelous Martin Gardner wrote a similar column in that great magazine, "Mathematical Games" for thirty years. Math <u>is</u> fun, indeed a great game. I remember reading Martin Gardner's column every month as a child, and finding great pleasure in wherever he might choose to wander that issue. Math? Pleasure? Too often these days, my students moan at the mention of math. And they're bad at it. My twin motives for making Math Magic Blocks have been to get kids to like math and, once they "get a liking for it" (see Leonardo, above), get better at it.

The journey started in 1982 when, as they say, "in a flash of insight", I came up with the concept of Math Magic Blocks. For many years, the prototype sat gathering dust on a top shelf in my classroom. I'd take it out to show someone every few years, but never thought about doing more until I demonstrated it to some hotshot chemistry teachers at an Advanced Placement workshop. They loved it, said I HAD to manufacture it because–*it would help their high school juniors and seniors understand the math of chemistry better.* With that nudge I finally started the arduous journey leading to what you see before you. Whenever I questioned whether it was worth all the work, my drive would be renewed by interactions like this:

One day I was working with three students, who were building a solar powered electric car. They needed to cut a 75 centimeter stick into two equal pieces and were discussing where to draw the cut line. The following conversation occurred:

TEACHER: "What is half of seventy five?"
(long silence)
TEACHER: "Well, what is half of seventy?"
(another long silence, as the students tried in vain to work the problem out in their heads)
TEACHER: "OK, what is half of eighty?"
(another not-quite-so-long pause),

STUDENT A (tentative): "Fifty?"
STUDENT B (with the eagerness that comes with certainty): "Sixty!"
At this point, STUDENT C pulled out his cell phone to work out the problem.
These students were sophomores, with Algebra I under their belts, and eight months of Geometry fresh in their minds.

Contrast this with my quizzing a middle school student working at her mother's strawberry stand at the local farmer's market this morning. When I asked her what half of seventy five was, she nailed it instantly: "Thirty seven fifty."

These anecdotes suggest that the middle schooler, who daily manipulates money, has a greater facility at manipulating numbers than my sophomores, with two more years of mathematics education. But this is not an isolated case. I recently wrote this on the board:

$$\frac{1}{8} \times 8 = ?$$

My class of thirty high school sophomores (many of whom were presently taking Algebra II) were stumped. The telling silence if I ask them to find one and three quarters on a ruler, or "What is one half of one third?" suggests something is seriously wrong. Their math curriculum does not lack for units teaching just what is needed to solve each of these problems. They are taught the necessary algorithms, and may even earn impressive scores on the Unit Test, proving mastery of the material. But their ability to pull that problem-solving ability out of their mental tool box has a short half-life. A math manipulative like Math Magic Blocks can address this issue in a memorable way. While it might seem that allowing your students to "play" with Math Magic Blocks is not the most efficient way to raise test scores, the heightened level of student interest in math, and the long term learning more likely to result suggests greater long term benefit.

QUICK START

INSTRUCTIONS FOR SETTING UP THE 2-PAN BALANCE

Assembly of the scale (as seen on box the scale comes in):

1. Press blue pedestal into blue base.
2. Snap yellow balance arm into top of pedestal. (The balance arm must be pushed down and wiggled to get it to snap onto the pedestal). .
3. Snap two white pan holders onto two pans. Take care not to break the tabs on the pans, snap the 3-pronged white plastic pieces on to the pans.
4. Hang pan holders from ends of yellow balance arm. The pans hang from the balance arm, preferably with the small pouring spout pointing out from the center, to facilitate loading and removing blocks.

Zeroing the Balance (important!)

5. With no weight in either pan, slide the blue plastic pieces in or out until the yellow arrow on the balance arm points precisely at the arrow on the blue pedestal. The balance has been modified to detect small differences in weight between the two pans. The balance is remarkably sensitive (for being so cheap), but it is important to VERY CAREFULLY adjust the blue sliders, with both pans empty, so the two arrows (on blue pedestal and yellow lever) line up *exactly*. Until you zero the balance, you may think you are demonstrating that 32 = 33, where 32 (= 4 × 8) and 33 (= 3 × 11). The combined weight of 3-block plus11-block actually is slightly heavier, but if the balance is not carefully zeroed, it may not appear so.
6. Balance is now ready to use. Place blocks in the pans to find what number combinations balance, thus are “equal”.

NOTE: The black blocks and the colored blocks comprise two separate lessons; they cannot be mixed. For example, a black 2-block and a black 3-block will not balance <u>any</u> combination of colored blocks.

Math Magic Blocks addresses many of the instructional standards some teachers are required to teach (see below). But I would suggest that, as an introduction, don’t tell your students anything about the blocks. In fact, the same goes for you, their teacher. I strongly recommend you stop reading right now, and just play for an hour with the blocks. (If you think you’ve got them all figured out in 10 minutes, I challenge you to prove the identity of the unknown Z-block :)

Cognitive Development

Advancing from Representational to Symbolic Number Sense

Math Magic Blocks help build a strong foundation in basic concepts. Some concepts are so basic that we adults may forget that at one point we did not understand even the concept of "number". A child's first strategy for knowing "How many?" is finger counting. Two blocks are twice as much as one block. But a 2-block is a single block, just as a 1-block is. In what sense is a 2-block "twice as much" as a 1- block? The black MMBs move the child from representational to symbolic understanding. All ten black blocks look almost identical. The only apparent difference is a different symbol on each block. Each symbol corresponds with a difference inside each block–its weight. Through manipulation of black blocks (which represent the whole numbers from 1 to 10) students learn kinesthetically, aurally, and visually, basic number sense. Specifically, they feel that 2 is more than 1 because the 2-block is heavier, and rattles louder, than the 1- block. They confirm the tactile sensation by placing the 1-block in one pan of the balance and the 2-block in the other. The relative size of the first ten integers can be confirmed using the balance.

While 1 + 9 = 10 seems self-evident to us, for a child understanding that equation requires first understanding what the symbols "1" and "9" mean, and then, understanding the idea that two symbols can be added together to get a different symbol, as opposed to 1 + 9 = 19, a more logical equation for beginners. We may not appreciate the mental work going on in a child's mind when she seems to be playing with random block combinations. Consider this child's struggle to confirm that 1+ 9 does in fact add up to 10: https://www.youtube.com/watch?v=J-zkzCnTjFg

Key Points Before Beginning Instruction

If appropriate, it is recommended to initially allow students to simply play with MMB's without any guidance. Appreciating MMB's first as a puzzle and a game fosters a positive attitude towards the blocks. One student, in his Dear Santa letter wrote:

Dear Santa: For Christmas, please send me a tablet, and Math Magic Blocks.

If students are having so much fun they don't realize they are learning, the lesson lasts longer.

RECOMMENDED GRADE LEVEL

Math Magic Blocks has been shown to be a highly engaging lesson/puzzle/game for students from Kindergarten through high school. At each grade, students are motivated to address different questions, based upon their level of mathematical proficiency. For example, while1st graders may ask, "What balances a 6-block?", Algebra II students may ask, "Since we're adding the weight of the 2-block and the10-block, how the heck does the total weight NOT add up to the weight of the 12-block, but instead balances the 20-block?”

ELEMENTARY MATH

When you do introduce the blocks to your kids, you may wish to first show them just the black blocks. Sit back and witness what a fine teacher is a student’s natural curiosity (see the quote by abbè Dimnet on box cover). Students can’t resist playing with the blocks, and, depending on their grade, will sooner or later realize that the black blocks are addition blocks. As for the X- and Y-blocks, resist the temptation to prompt the students to figure out their values. They won’t rest until they figure it out.

Depending on the grade level of your students, after students feel comfortable with the black blocks, bring out the colored blocks. Again, sit back and enjoy the show. Experience has shown that, when they say “These blocks don’t work!”, there is no need to prove them wrong. Just smile silently. Sooner or later, some student will stumble upon something like 2 + 3 is heavier than 5, but balances a 6 block –“ Hey, these blocks can do multiplication!”

ADVANCED MATHEMATICS CLASSES

Though high school students will not be learning how to add or multiply simple whole numbers with MMB’s, your more thoughtful students will be puzzled by how *adding* the weights of 2 blocks yields the *product* of the numbers on the blocks. Suffice it to say every student who has taken Algebra II has covered the math necessary to solve the puzzle. Along the way to their solving that mystery, the value of much of the math they’ve learned will become gratifyingly apparent.

Lessons using Math Magic Blocks

Math Magic Blocks can be used to teach many Content Standards, but I would encourage teachers to not focus too narrowly on this application. If you initially simply set out the MMB's and allow students to play with them, you may find applications that would not have occurred to you just scanning the Content Standards you are asked to cover. I gave sets of blocks to two teachers (Kindergarten and 3rd grade) at a charter school (who enjoy greater freedom with their course content). They used this more open-ended approach with inspiring results: https://www.youtube.com/watch?v=22UxR0qva6c

Of course, Math Magic Blocks do directly address a wide variety of Content Standards, including:

1) ADDITION: The black blocks demonstrate addition of simple whole numbers

2) MULTIPLICATION: The colored blocks demonstrate multiplication.

3) EQUALITY: When the pans balance, their contents are equal.

4) INEQUALITY: If the left pan is heavier than the right, LEFT≠RIGHT, LEFT > RIGHT, RIGHT < LEFT.

5) PRIME NUMBERS The colored blocks include every number, starting with 2, up to 12, including 11. Some of these blocks (4, 6, 8, 9, 10 and 12) can be balanced with two other blocks–they can be factored. Since 10 and 12 can both be factored, and every block larger than 11 also can be balanced with two or more other blocks, students may assume that 11 also can be balanced with other blocks. Their vain search for that special combination will "prime"them for a deeper understanding of prime numbers

6) PRIME FACTORIZATION: The student who uses this method has a competitive advantage over his/her classmates that compute products to answer each challenge in the "Factor Game" (see below).

7) BASIC ALGEBRA: THE CONCEPT OF AN UNKNOWN:
For the addition blocks, there are two unknown blocks: X and Y. They can be "solved for" by either a simple equality (Y balances 5) or an equation (2 + 3 balances Y).

For the multiplication blocks, there are three unknown blocks: X, Y & Z. They are unknowns, but students can determine the value of *one* of them by comparing it against known blocks (X balances 4). The identity of the Y- and Z- blocks can also be deduced by way of a logical syllogism. (See number 8) DEDUCTIVE REASONING, below.)

Of course, every block in the set is an unknown if it is placed number side down on the balance. This makes for a concrete lesson in algebraic equations *i.e.*:
Solve for X (the upside down block).
For black blocks: $2 + X = 6$.
For colored blocks: $2 \times X = 6$.
Students can quiz each other.

A funny story: I took a set of blocks to a friend's business and set them up on a table at the front of the office where half a dozen employees in the Sales Department were hard at work. As I started to explain the blocks to my friend, one by one the Sales Department folks stopped working and came over. They were curious about the X- Y- and Z-blocks and started coming up with theories as to their identity and testing their ideas with the balance. After forty five minutes my friend pointed out jokingly that all his workers were still on the clock, but doing nothing to make sales. In the end, they did figure out the identity of the Y-block and–finally–the Z-block. :)

8) DEDUCTIVE REASONING: There is no block or combination of blocks with number labels that will balance the Y-block or the Z-block. But using a process of logical deduction, it is possible to prove what those two unknowns are. This may prove a formidable challenge to your advanced students–and their teacher!

SUGGESTED ACTIVITIES
There are many activities which involve Math Magic Blocks. You are invited to make up games appropriate for your students, but here are a couple of ideas to inspire your own creativity:

MYSTERY BLOCK
One student places one block in one pan, one of which is upside down, to hide its number. Other students attempt to guess the identity of the mystery block by adding different combinations of blocks to the other pan. Two upside down blocks in one pan?–one equation, two unknowns.

THE FACTOR GAME
In this game, the teacher selects from the whole colored set two to four blocks, which are placed on one pan of the balance. Students then race to write a list of blocks (from those remaining) which will balance when placed in the other pan. Students put their solutions face down in a stack on the front desk. After all students have submitted their answer, the teacher checks submissions in order. The first correct answer wins. (The correct answer may be "IMPOSSIBLE", if the remaining blocks can't balance).

For example, most students will first multiply, but a more powerful method is prime factorization. For example, if 36 and 60 are placed on one pan, the product is 2160, which will balance with a 2, 2, 2, 9, and 30. Solving this longhand is tedious.

Using prime factorization is faster: $36 = 2\times2\times3\times3$ and $60 = 2\times2\times3\times5$. So the combined factors are $2\times\underline{2\times2\times2}\times3\times\underline{3\times3}\times5$. Since there aren't enough 2's or 3's, this can be reduced to $2\times8\times3\times9\times5$. As students catch on to the competitive advantage of prime factorization, they'll start using that method, making the game progressively faster.

ALIGNMENT WITH COMMON CORE

Though Math Magic Blocks can teach kids things not found on any standardized test (like, "Math is fun!"), they can be used effectively to teach the following Common Core State Standards:

KINDERGARTEN

Understand addition, and understand subtraction.
CCSS.Math.Content.K.OA.A1
Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
CCSS.Math.Content.K.OA.A.2
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

An infant's first understanding of the concept of number, a way of quantifying the world, comes upon realizing that not every object is unique, *viz* while one orange and one apple are two different things, two apples are *the same thing twice*. Only much later is this concrete representation abstracted to a symbol "2". All of the black blocks in MMB's are identical in shape, size and color. This teaser challenges the child, who cannot use visual cues (shape, size and color) to know how the blocks differ. Using the balance clearly shows there is a difference. The child cannot rely on her first way of understanding number (counting objects) to solve this puzzle inherent in MMB. Addition is a foundation for understanding multiplication, hence the black blocks in an MMB set.

The only difference among the black blocks is weight, but this is a difficult one to detect easily with the heavier blocks using the sense of touch. The balance allows the student to begin dealing with symbols as representations of exactly quantified reality–integers. Playing with the blocks to test which combinations balance is, in fact, an exercise in becoming adapt at understanding the symbolic representation of the first ten integers, and understanding how different integers combine to make larger integers. As well as addition, subtraction is made real . For example, if a student places a 2-block on one pan and a 5-block on the other, trial and error will lead to seeing that 2 + 3 = 5. This can be conceptualized as "There is a 2 and a 3 *inside* a 5." Rather than learning a meaningless (though easily memorized–and easily forgotten) algorithm, the student is prepared to understand the concept of subtraction as follows: "Since there is a 2 and a 3 inside a 5, if I take a 3 out of a 5, a 2 will be left." The student thus comes easily to grasp these building blocks of all future mathematical concepts.

For greater detail on this and other aspects of Math Magic Blocks, click here: https://www.youtube.com/watch?v=uLXw_a-lVfc

CCSS.Math.Content.K.OA.A.3
Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Black blocks provide concrete proof not only that 5 can be decomposed into two blocks (the 5-block balances the 2-block and the 3-block), but that there are multiple ways to decompose any number larger than 3, *e.g.* 2-block plus the 3-block balances the 4-block plus the 1-block: 2 + 3 = 4 + 1.

CCSS.Math.Content.K.OA.A.4
For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
The student sees in a concrete way that 10 can be decomposed (i.e. "is equal to") 1 & 9, 2 & 8, 3&7, and 4 & 6. In like fashion, it can be "proven" "right before the student's eyes" that 9 can be decomposed into 8 and 1, and so on.

CCSS.Math.Content.K.OA.A.5
Fluently add and subtract within 5.
In addition to addition, subtraction can be represented with Math Magic Blocks. Consider the case where a 5 is placed in one pan and 1 and 4 are placed in the other pan, but the 4 is placed number side down. As mentioned above (CCSS.Math.Content.K.OA.A1) if 1 + 4 =5, then in a sense, 1 and 4 are *inside* 5. The hidden block (4, in this case) is the other number, besides the visible 1, that is in 5. If you take that hidden number out of 5, you are left with 1. So the puzzle confronting the student is: what is the hidden number that, if taken away from 5, equals 1? Playing with black blocks 1-5 fosters fluency in addition and subtraction.

Grade 1

Represent and solve problems involving addition and subtraction.

CCSS.Math.Content.1.OA.A.1
Understand and apply properties of operations and the relationship between addition and subtraction.

The concept of addition is demonstrated by placing two black blocks (*i.e.* 2-block and 3-block) in one pan, and challenging the student to figure out which block will balance them when placed in the other pan.

The concept of subtraction is demonstrated by placing two black blocks (*i.e.* 2-block and 3-block) in one pan, and their sum (5-block) in the other pan, but turning one of the blocks (*i.e.* the 2-block upside down in its pan. The student is led to understand this setup as follows: "Five is equal to 3 plus another number. If you have 5 and you take away 3, what do you have left?"

Much time in math classes is spent eliciting the “right” answer, either orally, or on a test. This emphasis may result in right answers, but runs the risk of failing to develop any deeper understanding of the topic. Many students have memorized their addition tables, and can tell you the answer to 4 plus 5 = ? But they may be hard pressed to explain exactly what that very foundational word, “plus”, means. To better appreciate the internal work a child is doing when they don’t *appear* to be doing anything but sitting there, consider this running account, with commentary, of a teacher and his children using MMB’s, and the dad’s clever use of finger counting as part of the lesson:

> I first used the blocks with my 5 year old, Charlie, who will be starting school in August. Charlie and I used the blocks for about 20 or 30 minutes. His overall impression was that we were playing a game - not doing "school", which I think is exactly what you would want the kids to feel/say. I followed the lesson plan laid out for Kindergarten. Results went well –as predicted. When I asked Charlie how much "heavier is the 3 block than the 2 block". He answered, "8". I did my best not to chuckle.
>
> I asked him to place the 8 block with the 2 block to see if he was right (that the sides would balance). We had a discussion at that point (e.g., Why is 8 the wrong answer? What number might be a better fit?). After reasoning back and forth for a while, he decided "4" would be a better guess. We checked the scale. After further reasoning, he eventually reached the correct conclusion about the blocks by yelling "The smaller numbers need a "1"! (because we did 2 vs. 3, 3 vs. 4, 4 vs. 5 etc). At one point we placed the 8 on the scale (face up) and I placed the 9 on the scale (face down). As that side of the scale went down, I asked him, "You know that side is an 8. What do you think the other block is, Charlie?" And he answered "9" correctly. I asked him why. He said, "Because that side is heavier. So I think it's more than 8 and the smaller numbers need a 1". Then we added the 1 block to 8 to verify the correct answer.
>
> I then tried increasing the difficulty. On the scale I placed the 8, face up, and a 10 face down on the other scale. I asked him what he thought the heavier block was . . .and predictably, he said "9" again. But when we added the 1 block to the 8, he saw that it wasn't enough to create a balance. I began to teach him that "adding 1" wasn't going to be enough sometimes. I revealed that the other block was a 10, and that the "8" needed a "2" to create a balance.
>
> To teach the concept more concretely, we traced his hands (showing all ten fingers) on paper (giving him the ability to count to 10 without being forced to use his actual hands each time). I had him cover up eight fingers (representing the lighter block) on the paper and asked him, “How many fingers are left?” He said "2", and then we tested the balance again by adding the 2 block to the 8 to see if it equaled 10.
>
> To practice at a simpler level, we focused on just one hand on the paper (5 digits). I placed a 5 block, face up, on a scale. Then I placed the number 2 block, face up, on the other. We talked about which was heavier and why (e.g., 5 is a

bigger number so it was heavier). Then I had him figure out what block was needed to go with the 2 block to make a balance with the 5 block. He took the paper. Covered two fingers and determined how many were left over (three). We then tested it out on the scale. We then kept the 5 block on one side and put the 4 block on the other. Same routine. He used the hand on the paper (covered up 4 fingers to determine what was needed to add up to 5) and used the scale to verify his answer.

Using the ten-fingers drawing was an important aid to understanding: "Charlie, the 9 block is on this side and the 4 block is on the other . What do you think we need to add to the 4 to balance?" Charlie: "6". Dad: "Ok, test it out!" Then at that point, Charlie sees he's wrong as the 9-side of the scale flies up in the air. So Charlie was trained to go to the hands drawn on the paper. Cover up one finger (starting with 9 fingers, not 10). And then cover up an additional 4 to see what was left (check with 5 block to confirm).

Overall I think the experience was great! No doubt that Charlie had a good time. My opinion is that the blocks REALLY were helpful in making predictions / checking for correct math, and understanding addition and subtraction at a deeper level.

I also worked with my recently graduated 2nd grader, Bella, on the addition lesson plan. We obviously didn't need to draw hands on a paper , as she could do the math in her head. Here are a couple things I'll share about that experience: Bella held the 8 block & 9 block (both face down). I asked her, as the lesson plan stated, which was heavier. She said (predictably) that she couldn't tell. We discussed why that might be, etc. We placed the blocks face down on the scale. Showing that one block was heavier than the other. I then pointed to the heavier block and told her it was "9". I asked her what she thought the other was. She responded "5"! Well. This was a slightly disappointing prediction (father/teacher pride kicked in). But it presented an opportunity to have a great teaching moment! (e.g., We talked about why it was difficult to figure out which was heavier - not only that they were heavy, but since they weighed about the same it meant that they were likely CLOSE in number as well).

I then asked, "Bella, you said they were probably close in number. But when I told you this block was 9. you told me the other block was probably 5. Bella, are those close in number? She immediately saw that her guess wasn't very logical. And immediately responded that the other block must be an 8! Without turning the 8 block over . I told her to test it out on the scale. (she added "1" to the 8 proving that her prediction was correct). I knew Bella was having a blast. When I made the math more advanced, I could tell she was really struggling to figure out what to do (which is a very common experience for her–math is her Achilles heal).

So eventually I asked her, "Do you want a hint?" She responded, "No . Give me more time". I thought that was just a terrific (and surprising) response!

The fact that she wanted no assistance showed she had a love for the blocks. In addition, the next day, she asked if we could "play the blocks again".

As indicated above, a powerful strategy is the use of the "mystery block"–a block whose number is facing down which taps into a child's innate curiosity. This leads to a guessing game format, with the child guided to guess what the mystery block is, explain why, and then confirm their guess, or see that their guess was wrong, and learn why. The instructions contained here are highly detailed. Some mentors may appreciate this guidance. If you feel you can guide the child to a lesson's goal using your own strategy, please do so.

CCSS.Math.Content.1.OA.B.3
Apply properties of operations as strategies to add and subtract.
Though the terms "commutative and associative" sound rather intimidating, these properties of addition are made totally obvious by any all-blocks-facing-up display. For example, If 8 + 2 = 10 is known, then 2 + 8 = 10 is also known. (Commutative property of addition.)
To add 2 + 3 + 5, the second two numbers can be added to make a ten:
2 + 3 + 5 = 2 + (3 + 5) = 2 + 8 = 10. (Associative property of addition.)

CCSS.Math.Content.1.OA.B.4
Understand subtraction as an unknown-addend problem.
For example to understand what is going on in a problem like 9 - 4 = ? place the 9-block in one pan and the 4-block and the X-block in the other pan. Lead the student to understand that the 9-block has two blocks "inside" it–a 4-block, and another block–the mystery X-block. As you say, "If we take the 4-block out of the 9-block what part of the 9-block is still left? Can you guess the mystery block?" At this point, take the 4-block and the 9-block out of the pans, leaving the X-block alone, unbalanced. Challenge the student to find a block that will balance it in the other pan. Once they hit upon the 5-block, take out the X-block and add the 4-block and 9-block to make the pans balance. Then, picking up the 4-block as you say, "9 take away 4 leaves 5". See the above, more detailed explanation of this lesson following **CCSS.Math.Content.1.OA.A.1** above.

CCSS.Math.Content.1.OA.C.5
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
Work with addition and subtraction equations.
As the student manipulates the black blocks, ask them to say aloud what they are doing.

CCSS.Math.Content.1.OA.D.7
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
The balance will reveal which of these equations (and all others) are true and which are false. You may wish to have the student write the equation, predict True or False, and then use the balance to confirm their prediction.

CCSS.Math.Content.1.OA.D.8
Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.
Do the same procedure as for **CCSS.Math.Content.1.OA.D.7**, above.

Grade 2

Represent and solve problems involving addition and subtraction.

CCSS.Math.Content.2.OA.B.2
By end of Grade 2, know from memory all sums of two one-digit numbers.
Work with equal groups of objects to gain foundations for multiplication.
Allowing students to play with the blocks will help achieve addition competency. If you wish a more direct instruction modality, work one-on-one with the student, or in small groups, asking students to write down equations, guess the answers, and then see the correct results as shown by the balance.

CCSS.Math.Content.2.OA.C.3
Write an equation to express an even number as a sum of two equal addends.
This standard can be done if you have two sets of Math Magic Blocks.

CCSS.Math.Content.2.OA.C.4
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

For this lesson, use all blocks face down or, alternatively, just use identical objects *i.e* pennies,. Set up an array of 2 columns and 3 rows like this:

■ ■
■ ■
■ ■

Ask the student to count aloud to find out how many objects there are, and then write the equations showing this:

3 + 3 = 6

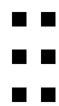
Continue in like fashion up to “5 rows and 5 columns”, but note that for all arrays where rows and columns are both odd numbers, it will not be possible to “express the total as a sum of equal addends.”, because the product of two odd numbers is always odd.

Grade 3

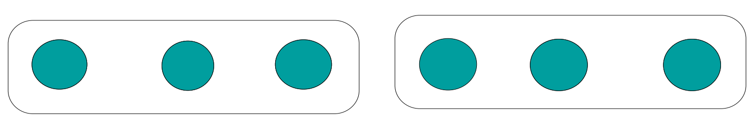
Represent and solve problems involving multiplication and division.

CCSS.Math.Content.3.OA.A.1
Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

For this standard, it is finally time to use the colored blocks! See standard **CCSS.Math.Content.2.OA.C.4** for Grade 2, above, for the addition lesson which precedes this multiplication lesson. This activity, which starts out like the addition lesson, that used the black blocks, uses only the colored blocks. To avoid confusion, move black blocks out of reach. Teacher arranges 6 blocks as an array of two columns, three rows:

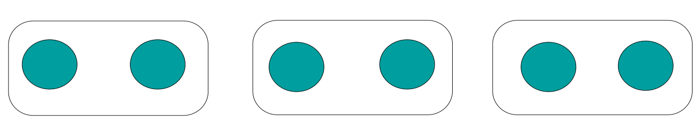


Teacher asks student to count up how many blocks there are (six). Teacher then points out to student that there are two columns (up and down), and three rows (sideways). Teacher points out there is a Magic Calculator that can answer the question "How many blocks in all?" without counting the blocks. Teacher asks the student to place a colored 2-block (the number of columns) in one pan, and a colored 3-block (rows) in the same pan. Student is then challenged to place a single block in the other pan that balances. Being used to addition, student may try a 5-block. It won't balance. Teacher offers encouragement until student tries a 6-block. Teacher points out that six is the number of blocks in the arrangement. Magic! Student first writes out this equation: 3 + 3 = 6 thus "expressing the total as a sum of equal addends". Next the student writes down this equation: 2 × 3 = 6. Student then copies this drawing:



Teacher explains that the drawing shows that "two groups of 3 circles each make 6 total circles." Student then copies this drawing:

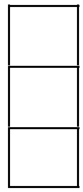
Student is asked to draw circle around each group of just 2 circles:



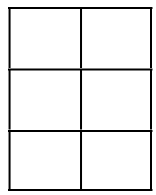
Teacher explains that this also has a total of six circles. This means that $2 \times 3 = 3 \times 2 = 6$ Student demonstrates the commutative property of multiplication for themselves by placing a2-block in one pan and a 6-block in the other, and then adding a 3-block to make it balance. They then are asked to place a 3 block in one pan and a 6-block in the other, which also makes it balance. Teacher then explains that "Multiplication is Speed Addition."

Teacher then challenges student to draw various arrays of "boxes", such as:

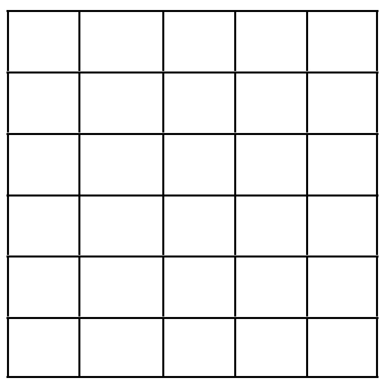
A stack of 3 boxes:



Two stacks of 3 boxes:



Five stacks of 6 boxes:



Student is guided to count the "total number of boxes", writing a check mark in each box as it is counted aloud, and writing the final count beside the drawing:

✔	✔	✔	✔	✔
✔	✔	✔	✔	✔
✔	✔	✔	✔	✔
✔	✔	✔	✔	✔
✔	✔	✔	✔	✔
✔	✔	✔	✔	✔

=30

If this seems tedious, that is the point. Student will see that it is possible to find the number of boxes by laboriously counting every one (addition), but there is a faster way: (multiplication). Teacher places a 6-block in one balance pan, saying “This is how many boxes are in each stack.” Teacher then asks student, “ How many stacks are there?”, and asks student to place that block (the 5-block) in the pan with the 6-block. Teacher then challenges student to find the block which, when placed in the other pan, balances (30-block). Magic! Student is then challenged to confirm that this process works for simple box arrays (i.e. two stacks of 3 boxes, see above). Visual inspection can determine without the need of counting every block.

Student is then shown a large box array:

Student is challenged to determine “How many boxes are there?”

At this point, depending on how well the student has grasped the concept, he/she may apply the previous method (laboriously counting, checking off each box), which is fine. It may take several exercises like this for the “light to go on”, and the student realizes all that work is not necessary. Whenever the “light goes on”, the student will be able to rapidly answer the “How many boxes?” question by placing two blocks in one pan and determining the block that balances in the other.

For larger numbers than 60, the Teacher proceeds as follows: Teacher places the 5-block and the 12-block in one pan, and demonstrates that the 60-block balances. Teacher then asks student to remove the 60-block, and replace it with two blocks that balance (*i.e.* a 2-block and

30- block). Teacher then discusses with student the idea that the 60-block can be represented by two blocks: the 2-block and the 30-block. Once the student grasps this concept, the teacher then removes the 30-block, and puts the 60-block back in with the 2-block. The student is guided to realize that a new very large number is represented in that pan. The student is then challenged to place two blocks in the other pan so it balances. Can the student make it balance *another* way? Possible solutions are: 10-block and 12-block, 8- block and 15-block, 5-block and 24-block. The concept is further solidified by challenging student to find two blocks that balance the pan with the 60-block and the 3-block.

Teacher then discusses this general principal with student: any two blocks in one pan can be balanced with a single block in the other pan that represents the total number of boxes in an array of stacked boxes. This is called "multiplication", which is faster than addition. The total number of boxes in an array is called the "product"

These concepts, originally established laboriously, can be written simply: (3 stacks) of (2 boxes in each stack) makes a total of 6 boxes (3) multiplied by (2) equals 6

$$3 \times 2 = 6$$

Student is asked to write the equation that represents each of these sentences:
"Five stacks of six boxes in each stack makes a total of thirty boxes." ($5 \times 6 = 30$)
"Five stacks of twelve boxes in each stack makes a total of sixty boxes." ($5 \times 12 = 60$)
"Ten stacks of twelve boxes in each stack makes a total of one hundred and twenty boxes." ($10 \times 12 = 120$)

Referring back to $5 \times 6 = 30$, teacher asks student to demonstrate this by placing appropriate blocks in balance pans. Teacher then asks student, "What does the 5-block stand for–the number of stacks, or the number of boxes in each stack?" According to the sentence just above, the correct answer is "the number of stacks". This answer is confirmed by referring back to the 5 by 6 array of boxes the student drew, and laboriously checked off (see above). Teacher then takes the student's 5 by 6 array drawing, and showing the student, rotates it 90 degrees, to make a "six stacks of five boxes in each stack." Student is guided to see that, since the number of boxes has not changed, two numbers can be multiplied in either order: $5 \times 6 = 6 \times 5$ This is called the commutative property of multiplication. Student is challenged , "Does the commutative property work for addition, too?" If student hesitates, three black blocks that balance are placed in the balance to confirm the answer is yes.

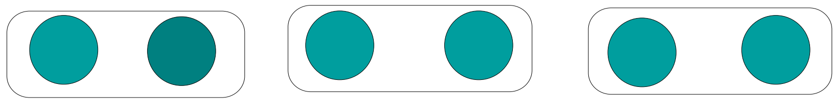
CCSS.Math.Content.3.OA.A.2

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Just as subtraction is the opposite of addition, so division is the opposite of multiplication. Referring back to **CCSS.Math.Content.2.OA.C.4** , where

$$2 \times 3 = 6$$

was represented as



This can be stated in words as How many shares of two each can six circles be partitioned into? If a 6-block is placed in one pan and a 2-bock and 3-block in the other, but the 3-block is upside down, then whatever the upside down block is, it is the answer to the question just asked. Using this basic idea, student can try other division problems of the form "12 divided into (though we usually say "divided by") shares of three blocks each will have how many shares?"

CCSS.Math.Content.3.OA.A.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?
Understand properties of multiplication and the relationship between multiplication and division.

This is a further expansion of **CCSS.Math.Content.3.OA.A.2** (above)

CCSS.Math.Content.3.OA.B.5
Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

This standard covers all three properties of multiplication: commutative, associative and distributive. Math Magic Blocks are a perfect aid to showing why these properties are true for multiplication. For commutative property, teacher places a 6-block in one pan and a 24-block in the other, and challenges the student to add one block so it balances (the 4-block). Teacher then removes all blocks and, for this second challenge, places the 4-block in one pan and the 24-block in the other. "Can you place just one block in a pan that makes it balance"? Yes, 6 × 4 = 24 and 4 × 6 = 24 also.

The associative property is demonstrated just as clearly: a 30-block is placed in one pan and the student is challenged to place *three* blocks in the other pan that will balance. Once the student has succeeded (by placing a 2-block, a 3-block and a 5-block all in the other pan), the teacher takes the 3-block and the 5-block out of the pan, and challenges the student to place a *single* block there that will balance (the 15-block). Teacher then leads student to say, and then write

$$2 \times 3 \times 5 = 30 = 2 \times (3 \times 5) = 2 \times 15 = 30.$$

The same procedure is then done to demonstrate that 2 and 3 can also be associated:

$2 \times 3 \times 5 = 30 = (2 \times 3) \times 5 = 6 \times 5 = 30$

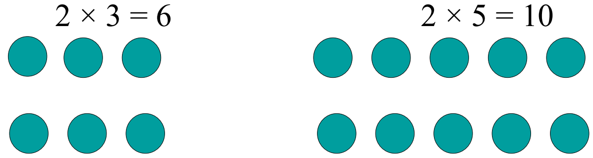
and that 2 and 5 can be associated:

$2 \times 5 \times 3 = 30 = (2 \times 5) \times 3 = 10 \times 3 = 30$

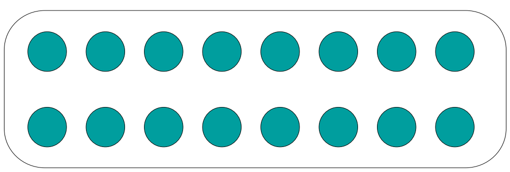
The distributive property can be demonstrated as follows: Student is first challenged to place a single block in the empty pan that will balance
a 2-block and a 3-block (= the 6-block), and
a 2-block and a 5-block (= the 10-block).

Then either by drawing, or placing counters (*e.g.* pennies or something similar), student writes creates this:

$2 \times 3 = 6$ $\qquad$ $2 \times 5 = 10$



Student then “Undistributes” the arrays by drawing a line, or moving the counters thusly:



Teacher then challenges student to show, with blocks, this array (2-block and 8-block in one pan balances the 16-block in the other pan). Teacher then leads student to understand that, if they’d started out with $2 \times 8 = 16$, they could have broken the problem down into

$$2 \times 3 = 6$$

and then added the result, 6, to

$$2 \times 5 = 10$$

This can all be represented in writing like this:

$$2 \times 3 = 6, \text{ and } 2 \times 5 = 10, \text{ so } 2 \times (3 + 5) = 2 \times 8 = 16$$

Teacher then leads student to understand that what you are doing is multiplying first, and then adding: 2 times 3 (= 6), and also multiplying 2 times 5 (= 10), and then adding those results together (6 + 10 = 16).

$$2 \times 3 = 6 \qquad 6$$
$$2 \times 5 = 10 \qquad \underline{+10}$$
$$16$$

But it is faster to do the adding first, then the multiplying:

$$2 \times (3 + 5) = 2 \times 8 = 16$$

Teacher explains that the parentheses mean you need to do the step *inside* the parentheses first. Teacher points out that if you don't do that step first, you get a different answer:

$2 \times (3 + 5) \neq 2 \times 3 + 5$, which would equal $6 + 5 = 11$ ☹

CCSS.Math.Content.3.OA.B.6
Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by8
Multiply and divide within 100.

A set of Math Magic Blocks weighs 10 pounds! Due to that fact that there are a limited number blocks in the box, the set does not happen to contain a 32-block. (On request though, I can make, and send to you any block you want smaller than the 60-block). There IS a 30-block, so the "unknown-factor problem) can be taught using the 3-block, 10-block and 30-block: Starting with

$$30 \div 10 = X$$

Student is challenged to figure out what the "mystery block" is when teacher sets up the balance with a 30-block in one pan and a 10-block, and a 3-block (which is upside down, so the number can't be seen) in the other pan. Student may know the answer right away, but if not, teacher removes the "mystery block", now allowing student to see it's number) and then challenges the student to find the answer by placing blocks, one and a time, in with the 10-block. (As there are two 3-blocks in the set, balance can be achieved with the *other* 3-block.) Once balance has been achieved, student can confirm the identity of the mystery block by removing the 30-block and 10-block, and then placing the mystery block in the other pan, noting it balances the 3-block already in the other pan, so

$$X = 3$$

The same procedure with different blocks can be employed to develop facility solving the unknown factor problem. Facility with multiplying and dividing within 100 is beyond the scope of the blocks in this set, but once the concept has been understood, developing facility with these larger numbers will be easier.

CCSS.Math.Content.3.OA.C.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.
This content standard summarizes the standards in **CCSS.Math.Content.3.OA.A.1** through **CCSS.Math.Content.3.OA.B.6.** Though the one-digit blocks do not *appear* to include a 7-block (but–surprise!– it IS in the box, labelled as the Y=block. More on this later.), all other "products of two one-digit numbers) can be demonstrated with the blocks in the set, facilitating student mastery of this standard.

CCSS.Math.Content.3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding
A sample of a two-step word problem whose solution can be demonstrated with Math Magic Blocks is:

> A fast food restaurant sells hamburgers for $4 and juice boxes for $2. If a group of 5 students order the hamburger and juice box combo each, what is the total cost of the group's order?

The student is challenged to demonstrate the steps involved in getting the answer which might be done as follows:
1) Using the black blocks, the students solves the problem, "What is the cost of the hamburger and juice box combo?" By placing a 4-block and a 2-block in one pan, and showing that it will balance with a 6-block in the other pan. Student then writes down the problem:

$$4 + 2 = X$$

and solves it:

$$4 + 2 = 6$$

Student then writes that information in a sentence: One hamburger and juice box combo costs $6.

Switching to using the colored blocks, student now demonstrates that 5 students each ordering $6 can be represented by placing the 5-block and the 6-block in one pan and balancing it with the 30-block in the other pan:

$$5 \times 6 = 30$$

Student then writes the final answer like this: "5 students each ordering $6 of food makes a total of $30 for the total cost of the group's order."

CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

All the sums on a standard addition table, such as this:

Addition Table

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

cannot be demonstrated by the black blocks. But the patterns in the table can easily be demonstrated. Remember Charlie, the Kindergartner, who, working with has dad, " eventually reached the correct conclusion about the blocks by yelling "The smaller numbers need a "1"!" Moving across the "1" row, one sees that each number on the orange heading row is 1 larger: 2 ➞ 3, 3 ➞ 4, 4 ➞ 5, etc. So the 1-block, added to any other block, balances with block 1 larger in the other pan. For the "2" row, the pattern is the same, but each number must be 2 larger. Your students may do finger counting to do the addition in these problems. Do not be alarmed! Recent research out of Stanford University (https://qz.com/668828/a-stanford-professor-says-counting-on-your-fingers-is-critical-to-understanding-math/) indicates that finger-counting, far from being banned in math lessons, is actually a useful step in developing mathematical competencies. From the article: ""Fingers are probably our most useful visual aid, critical to mathematical understanding, and brain development, that endures well into adulthood,"

A multiplication table to 10 is also beyond what Math Magic Blocks can fully demonstrate:

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

But MMB's can be a useful tool to build a foundation for understanding the overall structure of the table, which can facilitate later memorization of the whole table. For example, have the student start with a blank table:

×	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

and by place a 5-block in one pan. Placing a 2-bock in the same pan, and then determining which block in the other pan balances, and continuing with a 3-block, and 4-block, etc. the student can begin filling in the table thusly

×	1	2	3	4	5	6	7	8	9	10
1					5					
2					10					
3					15					
4					20					
5										
6					30					
7										
8										
9										
10										

The student can then more easily understand that "counting by tens" is going on. This makes it easier to fill in more numbers. Then, "counting by fives" completes the column:

×	1	2	3	4	5	6	7	8	9	10
1					5					
2					10					
3					15					
4					20					
5					25					
6					30					
7					35					
8					40					
9					45					
10					50					

Continuing with multiplying by two, block placement in the balance will confirm that the 2's column is just "counting by twos". Though, in the end, some memorization of the multiplication table may be needed, seeing overall patterns, revealed by use of Math Magic Blocks, will facilitate that achievement.

×	1	2	3	4	5	6	7	8	9	10
1		2			5					
2		4			10					
3		6			15					
4		8			20					
5		10			25					
6		12			30					
7		14			35					
8		16			40					
9		18			45					
10		20			50					

One simple insight, based on the **commutative property** (see **CCSS.Math.Content.3.OA.B.5** above) that will "cut the work is half" is the realization that one need only learn one half of the multiplication table as every product is repeated elsewhere in the chart. For example, since $3 \times 4 = 12$, and $4 \times 3 = 12$ also, once one learns the product of 3 and 4, one also has learned the product of 4 and 3.

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

In fact, the entire table is just a repeat of products along this diagonal line:

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Grade 4

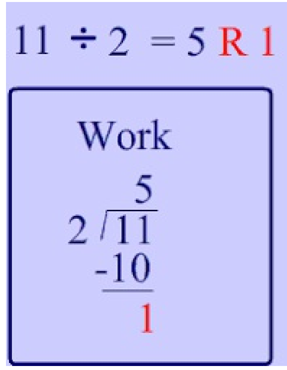
Use the four operations with whole numbers to solve problems.

CCSS.Math.Content.4.OA.A.1
Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

To address this standard, a 5-block and an 6-block are placed in one pan, and a 30-block is placed in the other. The student is then asked, 'Is this showing "five groups of six equals thirty, or that six groups of five equals thirty?"' If student responds that it is showing both, great–they got it! If they answer, for example, that it (only) shows that five groups of six equals thirty", take all the blocks out of the pans and ask the student place blocks in the pan to show "six groups of 5 equals thirty." The concept here is the **commutative property:**
CCSS.Math.Content.3.OA.B.5

CCSS.Math.Content.4.OA.A.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

This standard addresses the concept of the remainder in division. The concept involves both division (and, by extension, multiplication) AND addition. For example, in the problem "Divide eleven players into two equal teams", the division problem can be worked out as shown.



In this case, it would be messy, not to say downright ghoulish say the answer is "five and one-fifth players per team." In reality, one kid doesn't play. She remains on the sidelines, hence is the "remainder". To represent this with Math Magic Blocks, one can place the (colored) 2-block and 5-block in one pan and place challenge the student to place a block in the other pan that balances (10-block). Discuss with student that, if there are 10 kids, everyone plays in the game. But suppose there are more than 10 kids. Take out the 10-block and have the student put in the block that would mean *only one kid won't play* (the 11-block). Note the balance doesn't balance. Then replace the 11 block with the 12-block. Student notes that the balance falls down harder with the 12-block than with the 11-block–with 12 kids on the field, 2 kids don't get to play. The "remainder is 2". Now replace the 2-block with a 3-block and challenge the student to replace the 12 block with the smallest block so everyone gets to play (the 15-block). The goal of this activity is that the student understands that the "remainder" is not just a number one writes at the end of division problem. It can be thought of as how many kids are left over after a group is divided into teams of the same size. In this example, 11 kids were divided into 2 teams. Working out the problem shows that one kid doesn't play–the "remainder".

CCSS.Math.Content.4.OA.B.4
Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.
Generate and analyze patterns.

Math Magic Blocks are a perfect tool for determining factor pairs. If a 60-block is placed in one pan, and the student is challenged to find, and record, all the two-block combinations that will balance it in the other pan, he/she will write:

$2 \times 30 = 60$
$3 \times 20 = 60$
$4 \times 15 = 60$
$5 \times 12 = 60$
$6 \times 12 = 60$

An interesting insight stands in the wings of this "teachable moment" after the teacher explains the definition of "factor pair":

A factor pair is a pair of numbers that, when multiplied will result in the same product.

Ask the student if there is one factor pair they didn't write down. There is one more pair of numbers that, when multiplied, will result in a product of 60. Without (or with) assistance, the student will realize that 1 × 60 is another factor pair. At this point, a discussion of why there is no 1-block in the colored blocks *and why there IS a 1-block in the black blocks* will deepen the student's understanding of the mathematical operations, and the limitations of the Math Magic Blocks set. A related question, "What block is missing from BOTH the black and colored blocks?" is also worth addressing. The missing block referred to could be any number (*e.g.* 87) but such an answer is a trivial solution. A hint leading to the intriguing answer is: "If this block existed among the black blocks, it could be added to every balanced set of blocks in the pans and they would remain balanced, but it this block existed in the colored blocks, when it was added to either pan in a balanced set of blocks in the pans, it would make the *other* pan go down. The answer? A 0-block.

With only thirty six blocks in the box, I had to very carefully choose which ones to include. While I include 2, 4, 8, and 16 (which are 2^1, 2^2, 2^3, and 2^4,respectively, I stopped there, and did not continue on to 2^5 (32), even though 32 can be expressed as

$2^0 \times 2^5$ (that is 1×32)
<u>and</u> $2^1 \times 2^4$ (that is 2×16)
<u>and</u> $2^2 \times 2^3$ (that is 4×8).

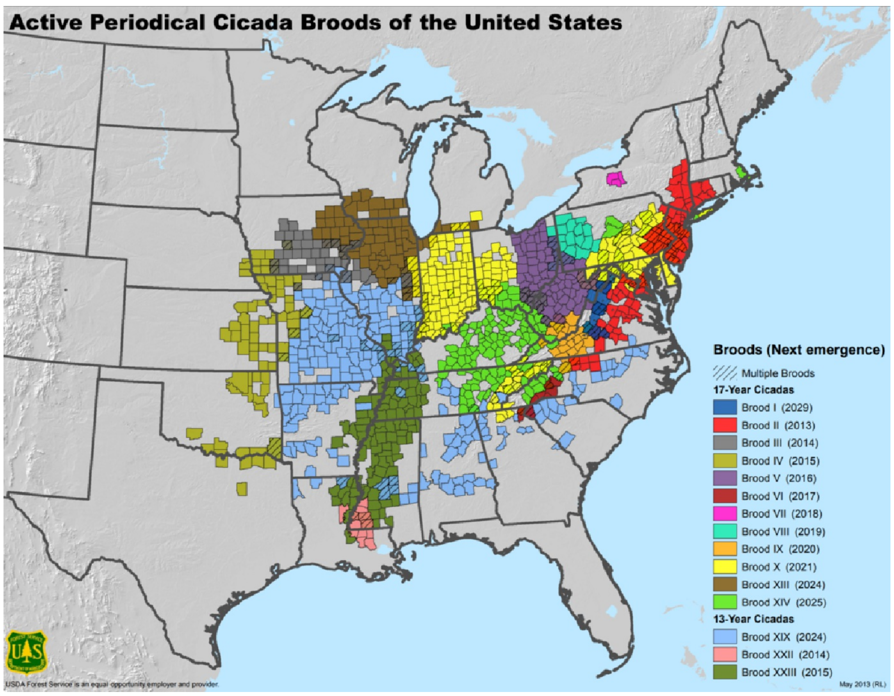
This makes for an interesting demonstration that exponents "multiply by adding":

$$2^1 \times 2^4 = 32 = 2^5 = 2^{(1 + 4)}$$

Ah!–SUCH a lovely number that 32! It was a tough decision, but, in the end, I decided that 30 would be included in the set, while 32, a "remainder", was not able to play. 30 cried out for inclusion for, among other reasons, it works in both "counting by tens" and in "counting by fives", it is the number of seconds in half a minute, and a dozen 30's make a full circle of degrees.

This is all by way of explaining two terms in the standard under discussion: **prime** and **composite**. Between them, these two categories contain every integer. Since "composite number" is defined as "A positive integer which is not prime", one could say that all positive integers are either composite or not composite. If I'd selected for Math Magic Blocks only prime numbers ("Prime" meaning a number that has no factors other than itself and 1) the game would be very frustrating–NOTHING would balance! But I did choose one number specifically to create an opportunity to teach the concept of prime number: the 11-block. It is book ended by to very composite numbers–10 and 12. An apt lesson to teach the concept of those numbers which are NOT composite is to put the any block that is a composite number face down in a pan and challenge the student to place two blocks in the other pan to balance. After several successes as this game, put the 11-block face down in a pan, stand back, and watch the frustration grow. But this is GOOD frustration, as it will lead on to a deeper appreciation for prime numbers than mere memorization.

One example: there are species of cicadas (commonly called "13-year cicadas") found in the Midwest with an intriguing life cycle that takes–can you imagine?–thirteen years to complete. There is also a species of 17-year cicadas. These insects spend all that time under the ground, only coming out, in huge numbers, at the very end of their life cycle, to mate. At that moment, they are an easy target for predators. There are no 14-, 15- or 16-year cicada species. One theory is that it would be disadvantageous for a cicada species to have a composite number of years in its life cycle. To understand this, consider the map of different cicada broods in the Eastern United States:



Notice there are many different 17-year cycles, and many areas where cicadas of one cycle border those of another cycle. If, say, there were a cicada with composite number of years in its life cycle, say a 16-year cicada, predators that had a 8-year life cycle that lived in a region where two different cicada broods bordered on each other would be able to prey upon cicadas every 8 years, instead of having a 16-year life cycle. Having 13 and 17 years for their life cycles would require any predators to synchronize their life cycle to this very long time period–an evolutionary feat nearly impossible. In fact, there are no predators with a life cycle that matches these two species.

Just one more example of the importance of prime numbers. In the age of the Internet, cryptography is a huge issue. Every time you submit a credit card for a purchase that is a "secure transaction", prime numbers make it nearly impossible to the bad guys to get your credit card number. This is because "semi-prime" numbers (a composite number that is the product of two primes) are easy to generate, and difficult to discover. But my lame explanation is not nearly as informative (or as fun!) As this: https://www.youtube.com/watch?v=56fa8Jz-FQQ.

Grade 5

Write and interpret numerical expressions.

CCSS.Math.Content.5.OA.A.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

The teacher can generate some challenges with numbers found within the Math Magic Blocks box. For example: Express the calculation "add 2 and 3 *and then* multiply by 4". First, student is challenged to write down the expression. The key to doing this correctly is the *"and then*" shown in italics). It indicates that there are two steps here that must be done **in order**: "add" *and then* "multiply". The student first writes down

$$2 + 3 \times 4 =$$

which gives the answer of 20. The teacher then discusses how this could be written differently , a discussion preparing the student for more advanced math (algebra) where it is customary to write an expression with the single number, or variable first, like this:

$$X(2 + 3)$$

rather than

$$(2 + 3)X$$

This "different" (flipped) way of writing would be

$$4 \times 2 + 3.$$

But as the student has already solved the problem and knows the answer is 20, a straightforward calculation would give 24 instead (4 × 2 = 8. 8 × 3 = 24.) The way out of this confusion is the use of parentheses, which tell the O.O.O. (Order Of Operations). The parentheses say "Do this part first". By writing the expression like this:

$$4 \times (2 + 3),$$

It makes clear that the first step, as the original problem stated, is to "add 2 and 3". Only then do you multiply by 4.

To make this more clear to the student, she is challenged to demonstrate using Math Magic Blocks the calculation requested. Before the student begins, point out that two operations (addition and multiplication) are both done in the problem. The question posed now to the student: "Should you use the black blocks or the colored blocks first?" The problem makes clear that the O.O.O. is black blocks first, then colored. At this point, the student manipulates the blocks to achieve the final answer (20), explaining each step aloud as they proceed.

2.1 Express a whole number in the range 2-50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as 2x2x2x3. Analyze patterns and relationships.

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Bruce Ratcliffe

PRAISE FOR MATH MAGIC BLOCKS

"I used the Math Magic Blocks in my 3rd grade class many times throughout the year to explore and reinforce concepts such as operations, measurement, and problem solving. My students enjoyed working with the MMBs and even the children who tend to shy away from math tasks were intrigued by the blocks. The MMBs were not only simple to manipulate, but also intriguing so that my students were motivated to think critically and work creatively. From a teacher's perspective, the Math Magic Blocks were a wonderful tool to energize my teaching and engage, excite, and challenge my students."

--Krista Tsutsui, 3rd grade teacher

"I brought home Math Magic Blocks for my 7 year old daughter and 5 year old son and let them create their own conclusions. Without any explanation of how the blocks worked or what to do with them, they spent hours working with the blocks in ways I could not have thought of. Since receiving them over six months ago, we still have not put them away. Math Magic blocks are part of the children's activities that continue to evolve in learning as they do. I recommend Math Magic Blocks for ANY parent who is focused on their children's learning and education."

--David Powers, parent of three